RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

THIRD YEAR

B.A./B.SC. FIFTH SEMESTER (July – December) 2014

Mid-Semester Examination, September 2014

Date : 15/09/2014 **PHYSICS (Honours)**

Time : 2 pm – 4 pm Paper : V Full Marks : 50

[Use a separate answer book for each group]

[*Answer **five questions** taking at least **one** from each group*]

Group – A

1. a) A system of N mass points (particles) moves under the action of applied forces $\vec{F}_i$ and forces of constraint $\vec{f}_i (i = 1, ..., N)$. If each particle is given a virtual displacement $\delta\vec{r}_i$, Show that the motion of the system is governed by D'Alembert's principle. State the basic assumption used in the derivation.

   Use this principle to obtain the equation of motion of a plane pendulum. [4+2]

   b) The point of support of a simple plane pendulum moves vertically upwards according to y = h(t), where h is some given function of time

      i) Find the Lagrangian, taking as generalised coordinates the angle θ the bob makes with the vertical.

      ii) Show that the pendulum behaves as a simple pendulum in a gravitational field, $g + \ddot{h}$. [2+2]

2. a) Show that if the Lagrangian of a system is not an explicit function of time, t, the energy function $E(q, \dot{q}, t)$ defined as $E(q, \dot{q}, t) = \Sigma \dot{q}_\alpha \frac{\partial L}{\partial \dot{q}_\alpha} - L$ is a constant of motion. Under what conditions is E the total mechanical energy of the system? [3]

   b) A particle of mass m moves in a central field with potential V(v). Write down the Lagrangian in terms of spherical coordinates (r, θ, ϕ).

      i) Find the momenta (p_r, p_θ, p_ϕ) conjugate to (r, θ, ϕ). Is any of them cnserved?

      ii) Find the Hamiltonian.

      iii) Write down the explicit Hamilton's equation of motion. [7]

Group – B

3. a) Write down the Lorentz transformation between two inertial frames with a relative velocity v along the common x axis. Show that the speed of light remains invariant from these two frames. [5]

   b) A rod of proper length l_0 oriented parallel to the x axis moves with velocity u along the x direction in an inertial frame S. What is the length measured by an observer in S' which is another inertial frame moving with velocity v along the common x axis? [5]

4. a) The origins of two inertial frames S and S' concide at $t = t' = 0$ and S' moves with a uniform velocity v along the positive x axis. There are clocks at rest in both S and S'. All clocks are synchronized at the initial moment. At an instant of time t find the position of the clock in S' that reads the same time as all the clocks at rest in S. [5]

   b) The frequency of light reflected from a moving mirror undergoes a Doppler shift because of the motion of the image. Find the Doppler shift of light reflected directly back from a mirror which is

approaching the observer with a velocity v, and show that it is the same as if the image were moving towards the observer a velocity $\frac{2v}{1+\frac{v^2}{c^2}}$. [5]

Group – C

5. Answers to each part question must be as brief as possible. Longer answers may lose credit. [5×2]
   a) Consider the electron double slit experiment with a monochromatic photon source to determine which slit the electron went through. If the intensity of the photon source is lowered keeping the frequency fixed, does the interference pattern reappear? What if the frequency of the photons is lowered, keeping the intensity fixed?
   b) In the photoelectric effect experiment, how does the photoelectric current change if the intensity of the light source is increased? Why?
   c) Can the earth's magnetic field be used to construct a Stern-Gerlach filter for spin one particles in any laboratory? Why?
   d) Why are observables in quantum mechanics chosen to be represented by Hermitian linear operators?
   e) If a system does not have stationary quantum states, what property of the Hamiltonian would you attribute this to?

6. In the flip-flop transitions of the ammonia molecule discussed in class, due to flipping of the position of the nitrogen atom relative to the three hydrogen atoms, the symmetry of the problem requires that the energy matrix had the properties $H_{11} = H_{22} = E_0$, $H_{12} = H_{21} = E'$. We now relax these symmetry requirements, dealing with an arbitrary 2×2 Hermitian Hamiltonian matrix H.
   a) Write down the time-dependent Schrödinger equation for the amplitudes of this system to be in either of the two states. [2]
   b) Obtain the energy levels for this system explicitly in terms of the elements of the H matrix. [4]
   c) How do these energy levels change with time? [4]

Group – D

7. a) What is Larmor Precession? [1]
   b) Calculate the expression for this precessional motion. [4]
   c) How do you get the idea of space quantization for this calculation? [2]
   d) Find the possible angles that $\vec{L}$ makes with z-axis when $\ell = 3$. [3]

8. a) Is it possible to do Stern Gerlach experiment on free electron to measure the spin magnetic moment? [1]
   b) Show that magnetic moment of an atom is $\mu = \left\{1+\frac{j(j+1)+s(s+1)-\ell(\ell+1)}{2j(j+1)}\right\}\sqrt{j(j+1)}\ \mu_B$. [4]
   c) What is L-S coupling? [2]
   d) Calculate the possible values of $\vec{L}.\vec{S}$ when L = 1, $S = 1/2$. [3]

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